Chapter 25. Complementary Angles

Exercise 25(A)

Solution 1(i):

$$\frac{\cos 22^\circ}{\sin 68^\circ} = \frac{\cos(90^\circ - 68^\circ)}{\sin 68^\circ} = \frac{\sin 68^\circ}{\sin 68^\circ} = 1$$

Solution 1(ii):

$$\frac{\tan 47^\circ}{\cot 43^\circ} = \frac{\tan(90^\circ - 43^\circ)}{\cot 43^\circ} = \frac{\cot 43^\circ}{\cot 43^\circ} = 1$$

Solution 1(iii):

$$\frac{\sec 75^\circ}{\operatorname{cosec} 15^\circ} = \frac{\sec(90^\circ - 15^\circ)}{\operatorname{cosec} 15^\circ} = \frac{\operatorname{cosec} 15^\circ}{\operatorname{cosec} 15^\circ} = 1$$

Solution 1(iv):

$$\frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ}$$
$$= \frac{\cos(90^\circ - 35^\circ)}{\sin 35^\circ} + \frac{\cot(90^\circ - 55^\circ)}{\tan 55^\circ}$$
$$= \frac{\sin 35^\circ}{\sin 35^\circ} + \frac{\tan 55^\circ}{\tan 55^\circ}$$
$$= 1 + 1$$
$$= 2$$

Solution 1(v):

$$\sin^2 40^\circ - \cos^2 50$$
$$= \sin^2 (90^\circ - 50^\circ) - \cos^2 50^\circ$$
$$= \cos^2 50^\circ - \cos^2 50^\circ$$
$$= 0$$

Solution 1(vi):

$$\sec^2 18^\circ - \operatorname{cosec}^2 72^\circ$$
$$= [\sec(90^\circ - 72^\circ)]^2 - \operatorname{cosec}^2 72^\circ$$
$$= \operatorname{cosec}^2 72^\circ - \operatorname{cosec}^2 72^\circ$$
$$= 0$$

Solution 1(vii):

$$\sin 15^\circ \cos 15^\circ - \cos 75^\circ \sin 75^\circ$$
$$= \sin(90^\circ - 75^\circ)\cos 15^\circ - \cos 75^\circ \sin(90^\circ - 15^\circ)$$
$$= \cos 75^\circ \cos 15^\circ - \cos 75^\circ \cos 15^\circ$$
$$= 0$$

Solution 1(viii):

$$\sin 42^\circ \sin 48^\circ - \cos 42^\circ \cos 48^\circ$$
$$= \sin(90^\circ - 48^\circ)\sin 48^\circ - \cos(90^\circ - 48^\circ)\cos 48^\circ$$
$$= \cos 48^\circ \sin 48^\circ - \sin 48^\circ \cos 48^\circ$$
$$= \cos 48^\circ \sin 48^\circ - \cos 48^\circ \sin 48^\circ$$
$$= 0$$

Solution 2(i):

$$\sin(90^\circ - A)\sin A - \cos(90^\circ - A)\cos A$$
$$= \cos A \sin A - \sin A \cos A$$
$$= 0$$

Solution 2(ii):

$$\sin^2 35^\circ - \cos^2 55^\circ$$
$$= \sin^2 35^\circ - [\cos(90^\circ - 35^\circ)]^2$$
$$= \sin^2 35^\circ - \sin^2 35^\circ$$
$$= 0$$

Solution 2(iii):

$$\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$$
$$= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} - 2$$
$$= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2$$
$$= 1 + 1 - 2$$
$$= 2 - 2$$
$$= 0$$

Solution 2(iv):

$$\frac{2\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$$
$$= \frac{2\tan(90^\circ - 37^\circ)}{\cot 37^\circ} - \frac{\cot(90^\circ - 10^\circ)}{\tan 10^\circ}$$
$$= \frac{2\cot 37^\circ}{\cot 37^\circ} - \frac{\tan 10^\circ}{\tan 10^\circ}$$
$$= 2 - 1$$
$$= 1$$

Solution 2(v):

$\cos^2 25° - \sin^2 65° - \tan^2 45°$

$= [\cos(90° - 65°)]^2 - \sin^2 65° - (\tan 45°)^2$

$= \sin^2 65° - \sin^2 65° - (1)^2$

$= 0 - 1$

$= -1$

Solution 2(vi):

$\left(\frac{\sin 77°}{\cos 13°}\right)^2 + \left(\frac{\cos 77°}{\sin 13°}\right)^2 - 2\cos^2 45°$

$= \left(\frac{\sin(90° - 13°)}{\cos 13°}\right)^2 + \left(\frac{\cos(90° - 13°)}{\sin 13°}\right)^2 - 2(\cos 45°)^2$

$= \left(\frac{\cos 13°}{\cos 13°}\right)^2 + \left(\frac{\sin 13°}{\sin 13°}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)^2$

$= (1)^2 + (1)^2 - 2 \times \frac{1}{2}$

$= 1 + 1 - 1$

$= 1$

Solution 3(i):

L.H.S.

$= \tan 10° \tan 15° \tan 75° \tan 80°$

$= \tan(90° - 80°) \tan(90° - 75°) \tan 75° \tan 80°$

$= \cot 80° \cot 75° \tan 75° \tan 80°$

$= (\cot 80° \tan 80°)(\cot 75° \tan 75°)$

$= (1)(1)$

$= 1$

= R.H.S.

Solution 3(ii):

L.H.S.

$= \sin 42° \sec 48° + \cos 42° \operatorname{cosec} 48°$

$= \sin(90° - 48°) \times \frac{1}{\cos 48°} + \cos(90° - 48°) \times \frac{1}{\sin 48°}$

$= \cos 48° \times \frac{1}{\cos 48°} + \sin 48° \times \frac{1}{\sin 48°}$

$= 1 + 1$

$= 2$

= R.H.S.

Solution 4:

(i) $\sin 59^\circ + \tan 63^\circ$

$= \sin(90 - 31)^\circ + \tan(90 - 27)^\circ$

$= \cos 31^\circ + \cot 27^\circ$

(ii) $\operatorname{cosec} 68^\circ + \cot 72^\circ$

$= \operatorname{cosec}(90 - 22)^\circ + \cot(90 - 18)^\circ$

$= \sec 22^\circ + \tan 18^\circ$

(iii) $\cos 74^\circ + \sec 67^\circ$

$= \cos(90 - 16)^\circ + \sec(90 - 23)^\circ$

$= \sin 16^\circ + \operatorname{cosec} 23^\circ$

Solution 5:

(i) We know that for a triangle ΔABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\frac{\angle B + \angle A}{2} = 90^\circ - \frac{\angle C}{2}$$

$$\sin\left(\frac{A+B}{2}\right) = \sin\left(90^\circ - \frac{C}{2}\right)$$

$$= \cos\left(\frac{C}{2}\right)$$

(ii) We know that for a triangle ΔABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A$$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\tan\left(\frac{B+C}{2}\right) = \tan\left(90^\circ - \frac{A}{2}\right)$$

$$= \cot\left(\frac{A}{2}\right)$$

Solution 6:

(i)

$$3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

$$= 3\frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ}$$

$$= 3\frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2$$

(ii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

$$= 3\cos(90^\circ - 10^\circ)\operatorname{cosec} 10^\circ + 2\cos(90^\circ - 31^\circ)\operatorname{cosec} 31^\circ$$

$$= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\sin 31^\circ \operatorname{cosec} 31^\circ$$

$$= 3 + 2 = 5$$

(iii) $\frac{\sin 80^0}{\cos 10^0} + \sin 59^\circ \sec 31^\circ$

$$= \frac{\sin\left(90^0 - 10^0\right)}{\cos 10^0} + \sin(90^\circ - 31^\circ)\sec 31^\circ$$

$$= \frac{\cos 10^0}{\cos 10^0} + \frac{\cos 31^0}{\cos 31^0}$$

$$= 1 + 1 = 2$$

(iv) $\tan(55^\circ - A) - \cot(35^\circ + A)$

$$= \tan[90^\circ - (35^\circ + A)] - \cot(35^\circ + A)$$

$$= \cot(35^\circ + A) - \cot(35^\circ + A)$$

$$= 0$$

(v) $\operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$

$$= \operatorname{cosec}[90^\circ - (25^\circ - A)] - \sec(25^\circ - A)$$

$$= \sec(25^\circ - A) - \sec(25^\circ - A)$$

$$= 0$$

(vi) $2\frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$

$$= 2\frac{\tan(90^\circ - 33^\circ)}{\cot 33^\circ} - \frac{\cot(90^\circ - 20^\circ)}{\tan 20^\circ} - \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$= 2\frac{\cot 33^\circ}{\cot 33^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1$$

$$= 2 - 1 - 1$$

$$= 0$$

(vii) $\dfrac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\dfrac{\sin^2 75^\circ}{\cos^2 15^\circ}$

$= \dfrac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2\dfrac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ}$

$= \dfrac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2\dfrac{\cos^2 15^\circ}{\cos^2 15^\circ}$

$= 1 - 2 = -1$

(viii) $\dfrac{\cos 70^\circ}{\sin 20^\circ} + \dfrac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$

$= \dfrac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \dfrac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8\left(\dfrac{1}{2}\right)^2$

$= \dfrac{\sin 20^\circ}{\sin 20^\circ} + \dfrac{\sin 31^\circ}{\sin 31^\circ} - 2$

$= 1 + 1 - 2 = 0$

(ix) $14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ$

$= 14\left(\dfrac{1}{2}\right) + 6\left(\dfrac{1}{2}\right) - 5(1)$

$= 7 + 3 - 5 = 5$

Solution 7:

Since ΔABC is a right-angled triangle, right-angled at B,

$A + C = 90^\circ$

$\therefore \dfrac{\sec A.\sin C - \tan A.\tan C}{\sin B}$

$= \dfrac{\sec A(90^\circ - C)\sin C - \tan(90^\circ - C)\tan C}{\sin 90^\circ}$

$= \dfrac{\operatorname{cosec} C \sin C - \cot C \tan C}{1}$

$= \dfrac{1}{\sin C} \times \sin C - \dfrac{1}{\tan C} \times \tan C$

$= 1 - 1$

$= 0$

Solution 8(i):

$\sin(90^\circ - 3A).\operatorname{cosec} 42^\circ = 1$

$\Rightarrow \sin(90^\circ - 3A) = \dfrac{1}{\operatorname{cosec} 42^\circ}$

$\Rightarrow \cos 3A = \dfrac{1}{\operatorname{cosec}(90^\circ - 48^\circ)}$

$\Rightarrow \cos 3A = \dfrac{1}{\sec 48^\circ}$

$\Rightarrow \cos 3A = \cos 48^\circ$

$\Rightarrow 3A = 48^\circ$

$\Rightarrow A = 16^\circ$

Solution 8(ii):

$$\cos(90^\circ - 3A).\sec 77^\circ = 1$$

$$\Rightarrow \cos(90^\circ - 3A) = \frac{1}{\sec 77^\circ}$$

$$\Rightarrow \sin 3A = \frac{1}{\sec(90^\circ - 12^\circ)}$$

$$\Rightarrow \sin 3A = \frac{1}{\operatorname{cosec} 12^\circ}$$

$$\Rightarrow \sin 3A = \sin 12^\circ$$

$$\Rightarrow 3A = 12^\circ$$

$$\Rightarrow A = 3^\circ$$