Chapter 25. Complementary Angles

Exercise 25(A)

Solution 1(i):

cos 22º _	$\cos(90^{\circ}-68^{\circ})$	sin68° _ 1
sin 68°	sin 68°	sin68° - 1

Solution 1(ii):

tan 47° _	tan(90° - 43°) _	$\frac{\cot 43^\circ}{-1}$
cot 43°	cot 43°	ot 43°

Solution 1(iii):

sec 75° _	sec(90° - 15°)	cosec15°_1
cosec 15°	cosec15°	cosec 15°

Solution 1(iv):

$\frac{\cos 55^{\circ}}{\cos 25^{\circ}} + \frac{\cot 35^{\circ}}{\cos 55^{\circ}}$
sin 35° tan 55°
$- \frac{\cos(90^\circ - 35^\circ)}{\cot(90^\circ - 55^\circ)}$
sin 35° tan 55°
_ <u>sin 35° tan 55°</u>
- sin 35° tan 55°
= 1 + 1
= 2

Solution 1(v):

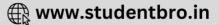
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sin^{2} 40^{\circ} - cos^{2} 50
= sin^{2} (90^{\circ} - 50^{\circ}) - cos^{2} 50^{\circ}
= cos^{2} 50^{\circ} - cos^{2} 50^{\circ}
= 0
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Solution 1(vi):

sec²18° - cosec²72° = [sec(90° - 72°)]² - cosec²72° = cosec²72° - cosec²72° = 0

Solution 1(vii):

sin 15° cos 15° - cos 75° sin 75° = sin(90° - 75°)cos 15° - cos 75° sin (90° - 15°) = cos 75° cos 15° - cos 75° cos 15° = 0



Solution 1(viii):

sin 42° sin 48° - cos 42° cos 48° = sin(90° - 48°)sin 48° - cos(90° - 48°)cos 48° = cos 48° sin 48° - sin 48° cos 48° = cos 48° sin 48° - cos 48° sin 48° = 0

Solution 2(i):

 $sin(90^\circ - A)sin A - cos(90^\circ - A)cos A$ = cos A sin A - sin A cos A = 0

Solution 2(ii):

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sin<sup>2</sup> 35° - cos<sup>2</sup> 55°
= sin<sup>2</sup> 35° - [cos(90° - 35°)]<sup>2</sup>
= sin<sup>2</sup> 35° - sin<sup>2</sup> 35°
= 0
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Solution 2(iii):

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\frac{\cot 54^{\circ}}{\tan 36^{\circ}} + \frac{\tan 20^{\circ}}{\cot 70^{\circ}} - 2
= \frac{\cot (90^{\circ} - 36^{\circ})}{\tan 36^{\circ}} + \frac{\tan (90^{\circ} - 70^{\circ})}{\cot 70^{\circ}} - 2
= \frac{\tan 36^{\circ}}{\tan 36^{\circ}} + \frac{\cot 70^{\circ}}{\cot 70^{\circ}} - 2
= 1 + 1 - 2
= 2 - 2
= 0
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Solution 2(iv):

$$\frac{2\tan 53^{\circ}}{\cot 37^{\circ}} - \frac{\cot 80^{\circ}}{\tan 10^{\circ}}$$

$$= \frac{2\tan (90^{\circ} - 37^{\circ})}{\cot 37^{\circ}} - \frac{\cot (90^{\circ} - 10^{\circ})}{\tan 10^{\circ}}$$

$$= \frac{2\cot 37^{\circ}}{\cot 37^{\circ}} - \frac{\tan 10^{\circ}}{\tan 10^{\circ}}$$

$$= 2 - 1$$

$$= 1$$

_ _ _





Solution 2(v):

$$\cos^{2} 25^{\circ} - \sin^{2} 65^{\circ} - \tan^{2} 45^{\circ}$$
$$= [\cos(90^{\circ} - 65^{\circ})]^{2} - \sin^{2} 65^{\circ} - (\tan 45^{\circ})^{2}$$
$$= \sin^{2} 65^{\circ} - \sin^{2} 65^{\circ} - (1)^{2}$$
$$= 0 - 1$$
$$= -1$$

Solution 2(vi):

$$\left(\frac{\sin 77^{\circ}}{\cos 13^{\circ}}\right)^{2} + \left(\frac{\cos 77^{\circ}}{\sin 13^{\circ}}\right)^{2} - 2\cos^{2} 45^{\circ}$$

$$= \left(\frac{\sin (90^{\circ} - 13^{\circ})}{\cos 13^{\circ}}\right)^{2} + \left(\frac{\cos (90^{\circ} - 13^{\circ})}{\sin 13^{\circ}}\right)^{2} - 2(\cos 45^{\circ})^{2}$$

$$= \left(\frac{\cos 13^{\circ}}{\cos 13^{\circ}}\right)^{2} + \left(\frac{\sin 13^{\circ}}{\sin 13^{\circ}}\right)^{2} - 2\left(\frac{1}{\sqrt{2}}\right)^{2}$$

$$= (1)^{2} + (1)^{2} - 2 \times \frac{1}{2}$$

$$= 1 + 1 - 1$$

$$= 1$$

Solution 3(i):

L.H.S.

- = tan 10° tan 15° tan 75° tan 80°
- = tan (90° 80°) tan (90° 75°) tan 75° tan 80°
- = cot 80° cot 75 ° tan 75° tan 80°
- = (cot 80° tan 80°)(cot 75° tan 75°)
- = (1)(1)
- = 1
- = R.H.S.

Solution 3(ii):

L.H.S. = $\sin 42^{\circ} \sec 48^{\circ} + \cos 42^{\circ} \csc 48^{\circ}$ = $\sin(90^{\circ} - 48^{\circ}) \times \frac{1}{\cos 48^{\circ}} + \cos(90^{\circ} - 48^{\circ}) \times \frac{1}{\sin 48^{\circ}}$ = $\cos 48^{\circ} \times \frac{1}{\cos 48^{\circ}} + \sin 48^{\circ} \times \frac{1}{\sin 48^{\circ}}$ = 1 + 1= 2 = R.H.S.



Solution 4:

(i) sin 59° + tan 63°
sin (90 - 31)° + tan (90 - 27)°
cos 31° + cot 27°
(ii) cos cos 68° + cot 72°
cos cos (90 - 22)° + cot (90 - 18)°
sec 22° + tan 18°
(iii) cos 74° + sec 67°
cos (90 - 16)° + sec (90 - 23)°
sin 16° + cos 23°

Solution 5:

(i) We know that for a triangle
$$\triangle ABC$$

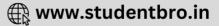
 $\angle A + \angle B + \angle C = 180^{\circ}$
 $\frac{\angle B + \angle A}{2} = 90^{\circ} - \frac{\angle C}{2}$
 $\sin\left(\frac{A+B}{2}\right) = \sin\left(90^{\circ} - \frac{C}{2}\right)$
 $= \cos\left(\frac{C}{2}\right)$

(ii) We know that for a triangle \triangle ABC

$$\angle_{A} + \angle_{B} + \angle_{C} = 180^{\circ}$$
$$\angle_{B} + \angle_{C} = 180^{\circ} - \angle_{A}$$

$$\frac{\angle B + \angle C}{2} = 90^{\circ} - \frac{\angle A}{2}$$
$$\tan\left(\frac{B + C}{2}\right) = \tan\left(90^{\circ} - \frac{A}{2}\right)$$
$$= \cot\left(\frac{A}{2}\right)$$





Solution 6:

(i)

$$3\frac{\sin 72^{\circ}}{\cos 18^{\circ}} - \frac{\sec 32^{\circ}}{\cos \sec 58^{\circ}}$$

$$= 3\frac{\sin(90^{\circ} - 18^{\circ})}{\cos 18^{\circ}} - \frac{\sec(90^{\circ} - 58^{\circ})}{\cos \sec 58^{\circ}}$$

$$= 3\frac{\cos 18^{\circ}}{\cos 18^{\circ}} - \frac{\csc 58^{\circ}}{\csc 58^{\circ}} = 3 - 1 = 2$$
(ii) $3\cos 80^{\circ} \csc 10^{\circ} + 2\cos 59^{\circ} \csc 21^{\circ}$

$$= 3\cos(90^{\circ} - 10^{\circ}) \csc 10^{\circ} + 2\cos(90^{\circ} - 31^{\circ}) \csc 31^{\circ}$$

$$= 3 + 2 = 5$$
(iii) $\frac{\sin 80^{\circ}}{\cos 10^{\circ}} + \sin 59^{\circ} \sec 31^{\circ}$

$$= \frac{\sin (90^{\circ} - 10^{\circ})}{\cos 10^{\circ}} + \sin 59^{\circ} \sec 31^{\circ}$$

$$= \frac{\sin (90^{\circ} - 10^{\circ})}{\cos 10^{\circ}} + \sin (90^{\circ} - 31^{\circ}) \sec 31^{\circ}$$

$$= \frac{\cos 10^{\circ}}{\cos 10^{\circ}} + \frac{\cos 31^{\circ}}{\cos 31^{\circ}}$$

$$= \frac{1 + 1 = 2}{(10^{\circ} \tan 10^{\circ})} + \frac{\cos 31^{\circ}}{\cos 31^{\circ}}$$

$$= 1 + 1 = 2$$
(iv) $\tan(55^{\circ} - A) - \cot(35^{\circ} + A)$

$$= \cot(35^{\circ} + A) - \cot(35^{\circ} + A)$$

$$= \cot(35^{\circ} + A) - \cot(35^{\circ} + A)$$

$$= \cot(35^{\circ} + A) - \cot(35^{\circ} + A)$$

$$= \cos ec[90^{\circ} - (25^{\circ} - A)] - \sec(25^{\circ} - A)$$

$$= \sec(25^{\circ} - A) - \sec(25^{\circ} - A)$$

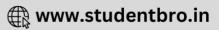
$$= \sec(25^{\circ} - A) - \sec(25^{\circ} - A)$$

$$= 2\frac{\tan(90^{\circ} - 33^{\circ})}{\cot 33^{\circ}} - \frac{\cot(70^{\circ} - 20^{\circ})}{\tan 20^{\circ}} - \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$= 2\frac{\cot 33^{\circ}}{\cot 33^{\circ}} - \frac{\tan 20^{\circ}}{\tan 20^{\circ}} - 1$$

$$= 2 - 1 - 1$$





$$\begin{aligned} \text{(vii)} \quad \frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\frac{\sin^2 75^\circ}{\cos^2 15^\circ} \\ &= \frac{\left[\cot(90^\circ - 49^\circ)\right]^2}{\tan^2 49^\circ} - 2\frac{\left[\sin(90^\circ - 15^\circ)\right]^2}{\cos^2 15^\circ} \\ &= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2\frac{\cos^2 15^\circ}{\cos^2 15^\circ} \\ &= 1 - 2 = -1 \\ \text{(viii)} \quad \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ \\ &= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2 \\ &= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2 \\ &= 1 + 1 - 2 = 0 \\ \text{(ix)} \quad 14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ \\ &= 14\left(\frac{1}{2}\right) + 6\left(\frac{1}{2}\right) - 5(1) \\ &= 7 + 3 - 5 = 5 \end{aligned}$$

Solution 7:

Since $\triangle ABC$ is a right-angled triangle, right-angled at B,

$$A + C = 90^{\circ}$$

$$\frac{\sec A \cdot \sin C - \tan A \cdot \tan C}{\sin B}$$

$$= \frac{\sec A(90^{\circ} - C) \sin C - \tan (90^{\circ} - C) \tan C}{\sin 90^{\circ}}$$

$$= \frac{\csc C \sin C - \cot C \tan C}{1}$$

$$= \frac{1}{\sin C} \times \sin C - \frac{1}{\tan C} \times \tan C$$

$$= 1 - 1$$

$$= 0$$

Solution 8(i):

$$\sin (90^{\circ} - 3A) \cdot \csc 42^{\circ} = 1$$

$$\Rightarrow \sin (90^{\circ} - 3A) = \frac{1}{\csc 42^{\circ}}$$

$$\Rightarrow \cos 3A = \frac{1}{\csc (90^{\circ} - 48^{\circ})}$$

$$\Rightarrow \cos 3A = \frac{1}{\sec 48^{\circ}}$$

$$\Rightarrow \cos 3A = \cos 48^{\circ}$$

$$\Rightarrow 3A = 48^{\circ}$$

$$\Rightarrow A = 16^{\circ}$$



Solution 8(ii):

$$\cos (90^{\circ} - 3A) \cdot \sec 77^{\circ} = 1$$

$$\Rightarrow \cos (90^{\circ} - 3A) = \frac{1}{\sec 77^{\circ}}$$

$$\Rightarrow \sin 3A = \frac{1}{\sec (90^{\circ} - 12^{\circ})}$$

$$\Rightarrow \sin 3A = \frac{1}{\csc e^{1}}$$

$$\Rightarrow \sin 3A = \sin 12^{\circ}$$

$$\Rightarrow 3A = 12^{\circ}$$

$$\Rightarrow A = 3^{\circ}$$



